DOI 10.2195/LJ_proc_supriyanto_de_201108_01

Fuzzy Multi-Objective Linear Programming and Simulation Approach to the Development of Valid and Realistic Master Production Schedule

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Abstract – Master production schedule (MPS) plays an important role in an integrated production planning system. It converts the strategic planning defined in a production plan into the tactical operation execution. The MPS is also known as a tool for top management to control over manufacture resources and becomes input of the downstream planning levels such as material requirement planning (MRP) and capacity requirement planning (CRP). Hence, inappropriate decision on the MPS development may lead to infeasible execution, which ultimately causes poor delivery performance. One must ensure that the proposed MPS is valid and realistic for implementation before it is released to real manufacturing system. In practice, where production environment is stochastic in nature, the development of MPS is no longer simple task. The varying processing time, random event such as machine failure is just some of the underlying causes of uncertainty that may be hardly addressed at planning stage so that in the end the valid and realistic MPS is tough to be realized. The MPS creation problem becomes even more sophisticated as decision makers try to consider multi-objectives; minimizing inventory, maximizing customer satisfaction, and maximizing resource utilization. This study attempts to propose a methodology for MPS creation which is able to deal with those obstacles. This approach takes into account uncertainty and makes trade off among conflicting multi-objectives at the same time. It incorporates fuzzy multi-objective linear programming (FMOLP) and discrete event simulation (DES) for MPS development.

Keyword: Master Production Schedule, Fuzzy Multi-Objective Linear Programming, ERP, Discrete Event Simulation, Production Planning

1 Introduction

According to American Production and Inventory Control Society (APICS), MPS is the declaration of what the company expects to manufacture in terms of configuration, quantities, and specific dates that drives MRP and other subsequent activities of a manufacturing company. Due to its vital role in production planning system, it is necessary to ensure that MPS is valid and realistic; otherwise the company may be unresponsive to customer needs or wasteful in its use of resources.

Proud argues that the real challenge of MPS creation is to balance available and requirement capacity [Pro99]. In this context, the viable solution may be easily found for deterministic capacity, however, it turns into intricate problem if both available and requirement capacities are stochastic or fuzzy. It is a fact that the available capacity of productive resource cannot be determined precisely due to unpredictable events such as unplanned breakdown, labor shortage, or material shortage. The requirement capacity for producing demand is also fuzzy because of varying processing time, setup time, or queue time.

In practice, it seems that this phenomenon is not taken into account by the modern ERP system. Rough-cut capacity planning (RCCP) function, which provides the productive resource profile for MPS, is not designed to deal with uncertainty. RCCP even considers only key or critical resources. Whereas, CRP function, which is expected to give a more detailed capacity check, is also neglecting uncertainty and even it performs only *infinite forward loading* instead of finite capacity analysis. It is in this sense that the current MPS system is limited to answering the question: Do we have a chance to meet the production plan? And do we have a chance to meet the master schedule?

Concerning to this issues, several studies have suggested a verification process to check the validity of tentative MPS. *Higgins et al.* have proposed a simulation of “What-if” analysis to be executed on the tentative MPS in order to find an optimum and realistic MPS [Hig92]. *Kochhar et al.* have developed a knowledge-based system approach, which is combining human expertise with computer computation, to achieve an accurate and realistic master schedule [Koc98]. *Heizer et al.* shared an idea about iterative planning process which allows a planner to check the validity of each planning process [Hei06].

In addition to the substitution of verification process, researchers also employ various advanced optimization technique in order to enhance MPS quality. For instance, *Vieira et al.* applied simulated annealing to solve MPS problem [Vie04]. This study reveals some drawbacks of simulated annealing such as overcoming local optimum. *Soares et al.* introduced new genetic algorithm structure for solving MPS problem [Soa08]. It formulates the fitness function which aims to minimize inventory level, maximize service level, minimize overtime and minimize inventory level below safety stock. At the end, *Vieira et al.* has compared genetic algorithms and simulated annealing for master production scheduling problems [Vie03].

The reviewed approaches have given worthy contribution to the development of MPS, especially to balance requirement capacity and available resource in effective manner while maintaining trade-off among the conflicting objectives. Only one thing needs to be improved is the underlying assumption of those approaches which ignore the capacity uncertainty. In order to bridge the gap, this research proposes a methodology, which employs FMOLP to model mathematical fuzzy objectives and constraints as well as DES to capture the dynamic behavior of manufacturing system and to verify the tentative MPS.

2 Iterative Production Planning

In order to ensure successful operation execution, the *valid* and *realistic* MPS must be developed [She03]. In this context, term *valid* means that the available capacity should be equal to the required capacity and the material due date is equal to the material need dates [Pro99]. Whereas, *realistic* means that the MPS must be feasible to implement and able to deliver the defined planning goal or decision maker (DM) targets. According to *Sheikh*, the development of actual scheduling is usually iterative, with a preliminary schedule being drawn up, checked for problems, and revised [She03]. After a schedule has been determined, some points should be checked: Does the schedule feasible to implement? Does the schedule meet the demand forecast? Does the schedule provide for flexibility and backups in case disturbance occurs? and so on. Problem in any one of these areas may force a revision of the schedule and a repeat of the previous step. The iterative process continues until all questions are answered and the planning goal is met.

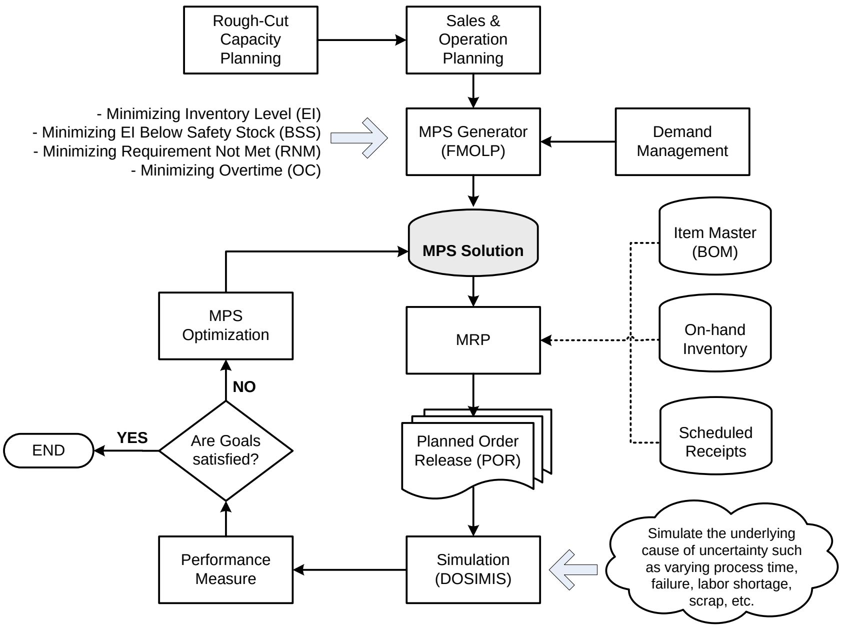

Figure 1: Schema of iterative MPS development

This study employs the same concept as the idea but using DES as a tool for the verification of master schedule. Figure 1 depicts a scheme for the creation of valid and realistic MPS proposed in this study. The demand management supplies all of the demand requirements and customer orders to MPS generator. Likewise, the sales and operation planning (SOP) indicates the operating constraints within which MPS function must work. MPS generator produces a tentative MPS solution that satisfies DM's target in terms of inventory level (EI), requirement not-met (RNM), inventory below safety Stock (BSS) and overcapacity (OC). Based on the given MPS, MRP function generates a set of planned order request (POR) by considering bill of material (BOM), on-hand inventory and scheduled receipts. POR is work document which contains information associated with production such as part number, quantitiy to be manufactured, work center, production start date, due date, and customer. To verify the validity of master schedule, the execution of PORs is simulated under various stochastic environments. The MPS is considered as valid and realistic only if the simulation output satisfies the defined DM's goals.

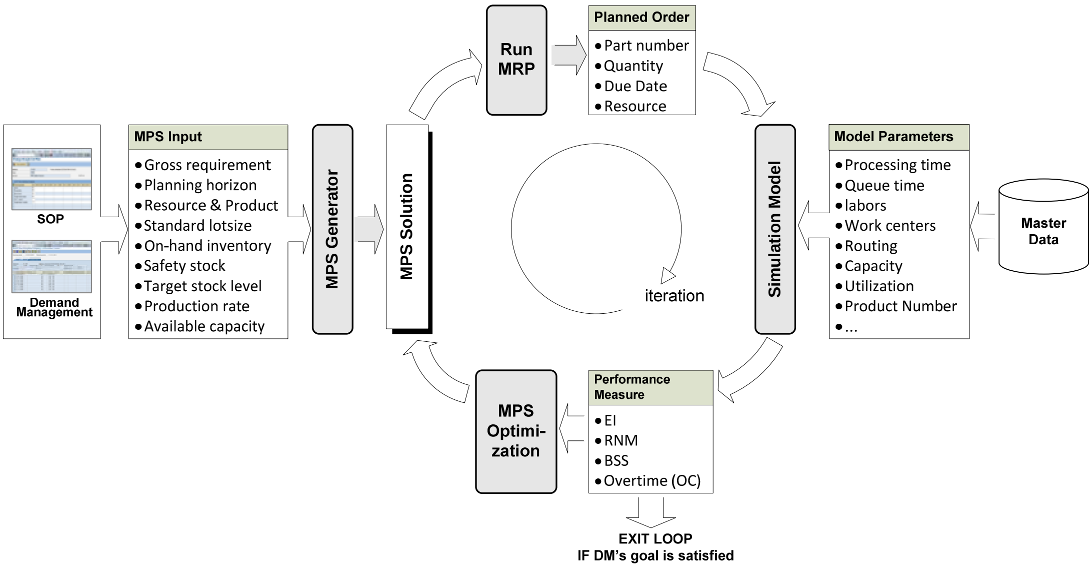

Figure 2: The information flow of MPS development

In order to realize the concept, an integrated planning system consisting of three commercial software (SAP, Matlab, and Dosimis-3) is developed. Figure 2 denotes the information flow of MPS development. To keep the consistency of planning logic, all production planning functions are executed in SAP except MPS. The MPS solution is generated separately in Matlab by adopting FMOLP technique whose formulation is explained in details in chapter 3. The input parameters (i.e. gross requirement, lot-size, safety stock, etc) described by *Vieira et al.* are automatically retrieved from SAP [Vie04]. Using those inputs, a tentative MPS is generated and subsequently entered into SAP as input of MRP. Simultaneously, Dosimis-3 retrieves master data (i.e. workstation, routing, processing time, capacity, etc.) from SAP to build simulation model. *Ziems et al.* have described step-by-step procedure to develop simulation model using Dosimis-3 [Zie93]. Then, the outcome of simulation is sent to Matlab. The key performance indicators are calculated and compared to the initial planning goal. If the DM's goals are not satisfied, the given MPS will be re-optimized and verified again. This process is repeated until the DM's goal is achieved.

3 MPS Generator

According to *Vieira et al.* [Vie03] and *Soares et al.* [Soa08], the MPS problem can be mathematically modeled as a mixed integer program as follow.

$$Minimize\, EI = \sum_{k=1}^{K} \left(\frac{\sum_{p=1}^{p} EI_{kp}}{TH} \right) \qquad Minimize\, OC = \sum_{r=1}^{R} \sum_{p=1}^{P} OC_{rp}$$

$$Minimize\,BSS = \frac{\sum_{k=1}^{K}\sum_{p=1}^{P} BSS_{kp}}{TH} \qquad Minimize\,RNM = \frac{\sum_{k=1}^{K}\sum_{p=1}^{P} RNM_{kp}}{TH}$$

Where,

$$CUH_{rp} = \sum_{k=1}^{K}\frac{(BS_{kp}BN_{kp}}{UR_{kr}} \qquad MPST_{kp} = \sum_{r=1}^{R} MPS_{kpr}$$

$$EI_{kp} = \max\lfloor 0,(MPS_{kp} + BI_{kp}) - GR_{kp} \rfloor \qquad MPS_{kpr} = BN_{kpr} \times BS_{kpr}$$

$$BSS_{kp} = \max\lfloor 0,(SS_{kp} - EI_{kp}) \rfloor$$

$$RNM_{kp} = \max\lfloor 0,(GR_{kp} - (MPST_{kp} + BI_{kp})) \rfloor$$

$$OC_{rp} = \max\lfloor 0,(CUH_{rp} - AC_{rp}) \rfloor$$

$$BI_{kp} = \begin{cases} OH_k & se(p=1) \\ EI_{k(p-1)} & se(p>1) \end{cases} \qquad (1)$$

When all goals are fuzzy and each of them has aspiration level Z^*_i *(i = 1,2,3,4),* the above crisp model is transformed into fuzzy statement as follow.

Find $x = \lfloor x_{111}, x_{112}, x_{113},..., x_{kpr} \rfloor$ to satisfy

$$\tilde{Z}_1(x) = \tilde{EI}(x) = \sum_{k=1}^{K}\left(\frac{\sum_{p=1}^{p} EI_{kp}}{TH}\right) <\sim Z_1^*$$

$$\tilde{Z}_2(x) = RN\tilde{M}(x) = \frac{\sum_{k=1}^{K}\sum_{p=1}^{P} RNM_{kp}}{TH} <\sim Z_2^*$$

$$\tilde{Z}_3(x) = BS\tilde{S}(x) = \frac{\sum_{k=1}^{K}\sum_{p=1}^{P} BSS_{kp}}{TH} <\sim Z_3^*$$

$$\tilde{Z}_4(x) = O\tilde{C}(x) = \sum_{r=1}^{R}\sum_{p=1}^{P} OC_{rp} <\sim Z_4^*$$

Subject to

$$g_2(x) = CUH_{rp} - AC_{rp} \le OL_{\max} \quad ; x_{kpr} \ge 0 \qquad (2)$$

Where OL_{max} represents maximum overtime allowed for each period. Assuming that Z^0_i and Z^1_i denote the value of the objective function Z_i *(i = 1,2,3)* such that the degrees of membership function is 0 or 1 respectively. The linear function of the membership function for objectives Z_1, Z_2, Z_3 is described as follow.

$$\mu_{Zi}(x) = \begin{cases} 0 & Z_i(x) \geq Z_i^1 \\ (Z_i^1 - Z_i(x))/(Z_i^1 - Z_i^0) & Z_i^0 \geq Z_i(x) \geq Z_i^1 \\ 1 & Z_i(x) \leq Z_i^0 \end{cases} \quad ; i = 1,2,3 \tag{3}$$

When a and b denotes the limit of the admissible violation of overcapacity, the triangular membership function for objective Z_4 is written as follow.

$$\mu_{Z4}(x) = \begin{cases} 1 - (Z_4^1 - Z_4(x))/a & Z_4^1 - a \leq Z_4(x) \leq Z_4^1 \\ 1 - (Z_4(x) - Z_4^1)/b & Z_4^1 \geq Z_4(x) \geq Z_4^1 + b \\ 0 & else \end{cases} \tag{4}$$

Using the convex fuzzy model proposed by *Bellman et al.* [Bel70] and *Sakawa* [Sak93], the equivalent crisp model for fuzzy formulation (eq.2) can be expressed as the following linear programming.

$$\max \quad w_1\mu_{Z1} + w_2\mu_{Z2} + w_3\mu_{Z3} + w_4\mu_{Z4}$$

subject to.

$$\mu_{Z1} \leq (Z_1^1 - Z_1(x))/(Z_1^1 - Z_1^0)$$

$$\mu_{Z2} \leq (Z_2^1 - Z_2(x))/(Z_2^1 - Z_2^0)$$

$$\mu_{Z3} \leq (Z_3^1 - Z_3(x))/(Z_3^1 - Z_3^0)$$

$$\mu_{Z4} \leq 1 - (Z_4(x) - Z_4^1)/b$$

$$\mu_{Z4} \leq 1 - (Z_4^1 - Z_4(x))/a$$

$$CUH_{rp} - AC_{rp} \leq OL_{\max}$$

$$w_1 + w_2 + w_3 + w_4 = 1$$

$$\mu_{Zi} \in [0,1] \;\; ; x_{kpr} \geq 0 \;\; ; i = 1,2,3,4 \;\; r = 1,2,3,... \;\; ; p = 1,2,3,... \tag{5}$$

Where w_i denotes the weighting coefficients that show the relative importance among the fuzzy objectives. The MPS solution x^* is obtained by solving the above crisp model using genetic algorithm.

4 MPS Optimization

*S*everal symptoms of mismanaged MPS can be identified from excessive unplanned overtime or overcapacity, extensively front – loaded capacity plan, excessive past due shop orders and increasing component inventory [She03, Pro99]. In this study, those symptoms will be used to justify the feasibility of tentative MPS. The optimization is carried out immediately as one of them is recognized during simulation. For this purpose, the fuzzy-based load leveling model is developed.

The principle of load leveling technique is simply to shift some quantities of product to be manufactured from the over-loaded resources to the under-loaded resources. However, this task becomes sophisticated as multiple resources, multiple products, and multiple periods are considered. One should determine how many quantities and which products should be shifted, from or to which resources, and from or to which period. Inappropriate decision may cost poor performance of other criteria. For instance, producing too much quantity in early planning period may decrease overtime during peak load but consequently lead to high inventory level. The task becomes more complex as the conflicting multi-objectives are involved. Fortunately, an advanced optimization technique such as FMOLP enables one to solve such intricate problem. The linear programming model for the load – leveling problem is formulated as follow.

$$\min \quad R\tilde{T}(y) = \max\left[0, OL - \sum_{k=1}^{K} \frac{y_k}{U_{ko}}\right] \qquad \min \quad D\tilde{I}(y) = \sum_{k=1}^{K} s_{ku} \times y_k$$

subject to.

$$g_1(y) = \sum_{k=1}^{K} \frac{y_k}{U_{ku}} \leq RC$$

$$y_k \leq x_{ko} \quad ; y_k \geq 0 \quad ; k = 1,2,\dots,K \tag{6}$$

Where $g_1(y)$ is constraint to ensure that resource R_u does not turn into overloaded after leveling. When the reduced capacities *[RT]* and the quantity of defect products *[DI]* are fuzzy, the equivalent crisp model for equation 6 can be written as follow.

$$\max \quad \beta_1 \mu_{RT} + \beta_2 \mu_{DI}$$

Subject to.

$$\mu_{RT} \leq \frac{RT(y) - RT^0}{RT^1 - RT^0} \qquad \mu_{DI} \leq \frac{DI(y) - DI^0}{DI^1 - DI^0}$$

$$g_1(y) = \sum_{k=1}^{K} \frac{y_k}{U_{ku}} \leq RC$$

$$y_k \leq x_{ko} \quad ; y_k \geq 0 \quad ; k = 1,2,\dots,K$$

$$\mu_{RT|DI} \in [0,1] \quad ; \beta_1 + \beta_2 = 1 \tag{7}$$

$[RT^0, RT^1]$ denote the lower and upper bound of aspiration level for the reduced load capacity, whereas $[DI^0, DI^1]$ indicate lower and upper bound of aspiration level for defect products. Those aspiration levels are obtained by solving the multi-objective problem as single objective using, each time, only one objective. Parameters $[\beta_1, \beta_2]$ indicate relative importance between fuzzy objectives and the degrees of membership function for each objective are as follow.

$$\mu_{RT}(y)=\begin{cases}1 & RT(y)\le RT^0\\ \dfrac{RT(y)-RT^0}{RT^1-RT^0} & RT^0<RT(y)<RT^1\\ 0 & RT(y)\ge RT^1\end{cases} \tag{8}$$

$$\mu_{DI}(y)=\begin{cases}1 & DI(y)\le DI^0\\ \dfrac{DI(y)-DI^0}{DI^1-DI^0} & DI^0<DI(y)<DI^1\\ 0 & DI(y)\ge DI^1\end{cases} \tag{9}$$

To obtain solution $y^*_k(k=1,2,..K)$, which is the quantities of products to be shifted to available resources, the above crisp equation is solved using genetic algorithm.

5 Study Case

In order to test and provide thorough illustration about the proposed methodology, a simplified production scenario is given and solved. Let's assume a typical manufacturing system which consists of four production resources (can be machines or production lines) and each of them has available capacity per period 8 hours. The detail MPS inputs and other production parameters are presented in Table 1.

Parameters	Notation	Value
• Number of products	K	4 (P-403, P-404, P-405, P-406)
• Number of production lines	R	4 (WCFXM01, WCFXM02, WCFXM03, WCFXM04)
• Planning horizon	P	20 days
• Standard lot size	BS_{kp}	10 unit for all products and all planning periods
• Average of production rate	UR_{rp}	Production rates for P-403, P-404, P-40, and P-405 are 100, 150, 200, and 400 units/hrs respectively
• Initial inventory	OH_k	Zero for all products
• Safety stock level	SS_{kp}	200 units for all products and all planning periods
• Gross requirement	GR_{kp}	Table 2
• Available capacity	AC_{rp}	8 hours/period for each resource
• Maximum overall overtime	AC_{tot}	4 hours during planning horizon
• Allowed maximum	OL_{max}	4 hours/ period for each resource
• Aspiration Levels	EI	$Z_1^0=50,\ \ Z_1^1=5250$
	RNM	$Z_2^0=0,\ \ Z_2^1=3200$
	BSS	$Z_3^0=10,\ \ Z_3^1=2000$
	OC	$a=b=Z_4^1=4$
• Goal Weighting Factors	w_i	$w_1=0.3,\ \ w_2=0.3,\ \ w_3=0.3,\ \ w_4=0.1$
	β_i	$\beta_1=0.7,\ \ \beta_2=0.3$
• Defect products (%)	s_{ku}	0% for all resources

Table 1: MPS Inputs and Production Parameters

Day	P- 403	P- 404	P- 405	P- 406	Day	P- 403	P- 404	P- 405	P- 406
1	1300	1250	1390	580	**11**	1400	1000	1610	680
2	930	1360	1360	530	**12**	340	1200	1460	690
3	1100	1660	1450	1650	**13**	420	320	1310	960
4	720	1490	1420	840	**14**	760	1440	1440	500
5	740	590	1270	230	**15**	880	1300	1070	280
6	1030	1520	1710	210	**16**	940	1030	1670	560
7	970	1570	1540	1710	**17**	1190	1010	1270	1360
8	1500	1500	1700	1810	**18**	720	840	1320	2040
9	850	1760	1360	1660	**19**	720	1240	1710	920
10	1610	1380	1240	140	**20**	580	1180	1530	320

Table 2: Gross Requirement

The tentative MPS Solution is obtained by substituting the corresponding parameters into MPS Generator which employs equation 5 as basis of algorithm. The given data result in MPS solution whose overall degree of satisfaction is achieved at μ_D = 0.8943 with the achievement level of each objective as follow.

$$EI^*\ (Z_1) = 1201;\ RNM^*(Z_2) = 160;\ BSS^*(Z_3) = 172;\ OC^*(Z_4) = 240\ min$$

These achievement levels are regarded as planning goal that must be achieved as execution. Figure 4 depict the profile of load capacity distribution produced by MPS generator. It shows that the requirement capacities are not distributed equally among the available resources. Some resources are over-loaded, some of them have been fully occupied, and the rest are partly occupied (under-loaded). If the historical information about the potential uncertainty in the manufacturing system is known or foreeable, the distribution of load capacity may be adjusted accordingly so that its unexpected impact can be minimized.

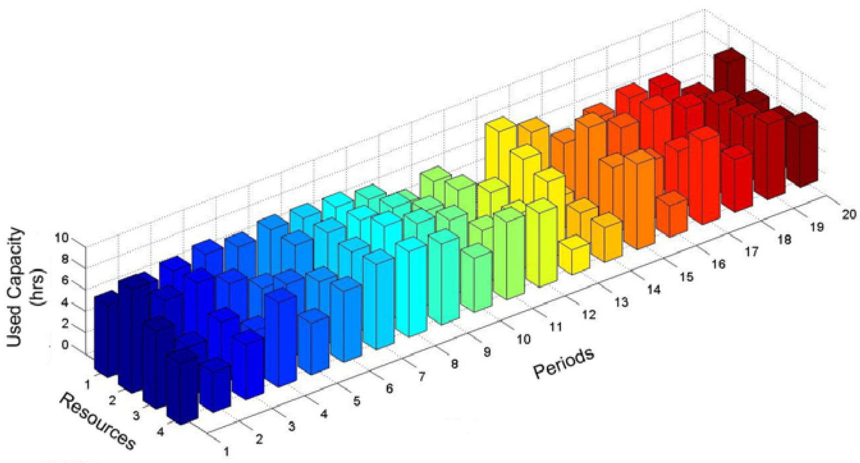

Figure 3 The profile of load capacity distribution

For the sake of simplicity, in this case the random machine breakdown (Table 3) and the varying processing time (Table 4) are chosen as the underlying cause of uncertainty. The processing time or production rate of each product is assumed as random normal distribution instead of constant numbers.

Failure Control	Prod. Resource	Stochastic Distribution	Availability	MTBF (min)		MTTT (min)	
				Mean	Deviation	Mean	Deviation
STO_1	WCFXM01	normal	98%	490	160	10.0	3.33
STO_2	WCFXM02	normal	96%	300	100	12.5	4.20
STO_3	WCFXM03	normal	97%	356	119	11.0	3.67
STO_4	WCFXM04	normal	95%	238	79.2	12.5	4.20

Table 3 Statistic of machine failure

Items	WCFXM01		WCFXM02		WCFXM03		WCFXM04	
	Mean	Std	Mean	Std	Mean	Std	Mean	Std
P-403	360	20	360	60	360	25	360	90
P-404	240	20	240	60	240	25	240	90
P-405	180	20	180	60	180	25	180	90
P-406	90	20	90	60	90	25	90	90

Table 4 Varying processing time (s/lot-size)

For analysis purpose, the simulation is carried out in several scenarios.

Scenario 1 No uncertainty or disturbance occurs during simulation

Scenario 2 Uncertainty is applied but no optimization is made

Scenario 3 Uncertainty is applied and adjustment is carried out by shifting overload to available resource at the same period, p.

Scenario 4 Uncertainty is applied and adjustment is carried out by shifting overload to available resource at period p and $p - 1$.

Scenario 5 Uncertainty is applied and adjustment is carried out by shifting overload to available resource at period p, $p - 1$ and $p - 2$.

Scenario 6 Uncertainty is applied and adjustment is carried out by shifting overload to available resource at period p, $p - 1$, $p - 2$ and $p - 3$.

The simulation outputs for all scenarios are summarized in Table 5. In this case, DM's objectives are the only indicator used to justify the planning validity. Master schedule which is able to deliver DM goals is considered as final solution.

No	Uncertainty	Scenario	Remark	EI	RNM	BSS	$OC(min)$
1	No	1	DM's Goal Satisfied	1201	160	172	240
2	Yes	2	Not Yet Optimized	1201	160	172	740
3	Yes	3	Optimization Stage 1	1201	160	172	304
4	Yes	4	Optimization Stage 2	1215	160	171	266
5	Yes	5	Optimization Stage 3	1261	160	169	88
6	Yes	6	Optimization Stage 4	1248	160	177	8

Table 5 Simulation Output

6 Evaluation

The first scenario is assumed as the ideal environment where no unexpected disturbance occurs. This simulation is meant to show that planning goals are easily obtained if there is no discrepancy between reality and schedule (all variables are deterministic). The uncertainty begins to be applied to the second scenario. The processing time is set random within certain deviation instead of constant and some machines suffer failures. One can see that DM’s goals can no longer be met, which is indicated by the significant increasing of overtime, from 240 to 740 min. It confirms that the tentative MPS turns into unrealistic due to uncertainty.

The third scenario, in which some load capacities are shifted to another resource at the same period, is able to almost half the unplanned overtime from 740 to 304 min without influencing other criteria. However, it does not meet DM’s goals yet. The fourth scenario, in which some overload capacities is leveled to the previous period, can drop successfully the unplanned overtime close to the planned overtime while other criteria seem not to be changed. The fifth scenario and sixth scenario indicate the similar results, where the overtime can be forced down significantly but it bring implication to the increase of inventory level. This emphasizes that in the MPS problem, the inventory level and overtime are conflicting objectives.

As a whole, the result exhibits that shifting loads to early planning period is able to get MPS to be more realistic. In practice, it implies that one should manufacture products as earlier as possible to anticipate peak load in the coming period if the production system is quite unstable. This strategy may increase inventory but keep service level high. Among the given results, the fourth scenario is probably one which is the most approximate to DM’s goals and may be chosen as final solution.

This simulation model considers a volatile manufacturing system which is indicated by frequent breakdown that occurs 1 – 2 times a day. In such volatile environment, relying only on the reactive action during execution phase can lead to the poor resource allocation and causes much unnecessary delay. Therefore, a realistic and optimum schedule, which takes into account the prediction of breakdown in its planning phase, is inevitable. In this context, one can see that the prediction of uncertainty plays important role in MPS creation using this methodology. Fail to acquire accurately information about the dynamic behavior of manufacturing system may make the planning output even getting worse.

7 Conclusion and Future Work

Uncertainty is one of root causes of the production planning inaccuracy. This study has proved that the underlying causes of uncertainty such as varying processing time, machine failure, etc have considerable implication on the MPS. Ignoring them can lead to incorrect decision and ultimately make poor production and delivery performance. Through simulation approach, this study has been able to recognize the effect of uncertainty and subsequently attempts to diminish its effect to the master schedule.

However, it is acknowledged that the given model may be only a simplified reality of manufacturing system. More sophisticated model should be developed to examine further the reliability of proposed methodology. The underlying cause of uncertainty needs to be extent as well; not only involving random breakdown and stochastic processing time but also other unpredictable occasions due to limited buffer space, labor shortage, material shortage, etc. Moreover, the applicability of this methodology in various environments with different grade of uncertainty may be worthwhile to be investigated as well. Note that this methodology may be not considered for stable environment in which uncertainty are insignificant and ignorable.

Variables

K	Total quantity of different products (SKU)
R	Total quantity of different productive resources
P	Total number of planning periods
TH	Total planning horizon
OH_k	Initial available inventory (on-hand), at the first scheduling period
GR_{kp}	Gross requirement for product k at period p
BS_{kp}	Standard lot size for product k at period p
NR_{kp}	Net requirement for product k at period p, considering infinity capacity
SS_{kp}	Safety inventory level for product k at period p
UR_{kp}	Production rate for product k at resource r (units per hour)
AC_{kp}	Available capacity, in hours, at resource r at period p
BN_{kpr}	Quantity of standard lot sizes needed for the production of the product k at resource r, at period p (number of lots)
MPS_{kpr}	Total quantity to be manufactured of product k at resource r, period p
$MPST_{kp}$	Total quantity to be manufactured of the product k at resource, at period p (considering all available resources)
BI_{kp}	Initial inventory level of the product k at period p
CUH_{rp}	Capacity used from the resource r at period p
CUP_{rp}	Percent rate obtained from the relation of the number of hours consumed from the resource r at period p, and the available number of hours to the same resource and period
GR_{kp}	Gross requirement for product k at period p
RM_{kp}	Total requirements met for product k at period p
RM_{kpr}	Total requirements met for product k at period p at resource r
RNM_{kp}	Requirements not met for product k at period p
R_o	Over-utilized or overload or overcapacity production resource
R_u	Under-utilized or under-load production resource
y_k	Quantity of product k to be shifted from R_o to R_u
x_{ko}	Planned quantity of product k to be manufactured in R_o
U_{ku}	Average of production rate R_u to produce product k (unit/hrs)
U_{ko}	Average of production rate R_o to produce product k (unit/hrs)

OL Actual overtime occurred in R_o
RC Remain capacities in R_u
s_{ku} Percentage of potential defect product occurred in R_u during operation execution
RT Amount of load capacity to be reduced from R_o (min)
DI Total number of defect products

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