

SAFE Merge: Merging Streams in Discrete Time Stochastic Flow Networks

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Merging stochastic flows is a central operation in discrete time queueing networks. Furmans' renewal-based merge provides an efficient analytical approximation, but can become inaccurate when the temporal structure of the input streams restricts the attainable arrival configurations and the merged process therefore deviates from a renewal process. This paper introduces the SAFE (Senger–Altmann–Furmans Extension) merge, an analytical extension for zero relative temporal offset that removes inadmissible forward recurrence time values before applying the original merge procedure. The method is evaluated against simulation and a numerical merge using the Wasserstein distance. The results show that SAFE remains close to the simulation benchmark for cases in which Furmans' approximation becomes inaccurate, while the numerical approach may require up to 10^8 samples to reach comparable accuracy.

[Keywords: discrete time, queueing theory, stochastic merge, open queueing networks, point process]

1 Introduction

Real-world systems rarely consist of a single sequential chain. Jobs may follow different processing paths and merge at shared resources, making open queueing networks a natural representation of such systems.

Discrete time queueing models are particularly suitable for material flow systems because they can directly incorporate empirical inter-arrival and service time distributions without assuming a specific distributional form. They also enable the computation of complete performance-measure distributions and corresponding quantiles, which are essential for service-level evaluation. Compared with simulation, analytical queueing models can reduce both modeling

and computational effort when evaluating alternative system configurations [1].

Accordingly, discrete time queueing models have been applied to a broad range of intralogistics systems, including transport and milk-run systems, multi-server resources, conveyor systems, order picking, and automated storage and retrieval systems [2, 3, 4, 5, 6].

However, the analysis becomes substantially more challenging when multiple flows merge, as illustrated in Figure 1. In discrete time, this requires the superposition of point processes while preserving their temporal characteristics, since the resulting merged arrival process directly determines the behavior of downstream queues.

For example, an automated storage and retrieval system can be represented in a discrete-time queueing network by two arrival streams, one for storage requests and one for retrieval requests, which are served by a common storage and retrieval machine. Depending on the upstream control and release processes, these streams may be temporally synchronized rather than independent. Furmans' [7] merge approximation represents the input streams by their marginal inter-arrival time distributions and approximates their superposition as a renewal process. If the streams are synchronized, however, their relative arrival positions are restricted and are not fully described by the marginal distributions alone. As a result, the renewal approximation may assign probability to merged inter-arrival times that cannot occur in the actual system.

However, while these limitations are identified qualitatively, the conditions under which the renewal approximation provides sufficient accuracy are not quantified. This paper addresses this gap by systematically investigating the validity range of the renewal approximation and extending Furmans' formulation by removing inadmissible probability values and subsequently renormalizing the resulting distribution in cases where the original approximation becomes

inaccurate. The proposed extension is formulated for zero relative temporal offset between the input processes, i.e., without an additional phase shift between the streams.

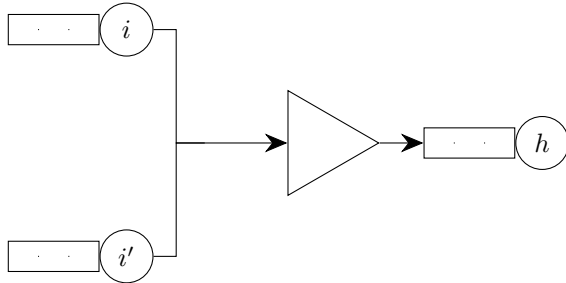


Figure 1: Merging of the flows from i to h and i' to h

2 Literature

The methodological basis is provided by Bruneel's treatment of general discrete time queueing systems [8]. Building on this foundation, Grassmann and Jain develop a numerical method for determining the waiting-time and idle-time distributions of an arithmetic $G/G/1$ queue, i.e., a single-server queue with generally distributed independent inter-arrival times and generally distributed service times on a discrete time grid [9]. Jain and Grassmann analyze the corresponding departure process, which is particularly relevant for decomposed flow-network models because the output of one queueing element often serves as the input of a downstream element [10]. Furmans and Zillus determine the distribution of the number of customers in a discrete time $G/G/1$ queue at arrival points, thereby supporting the modeling of independent buffer elements [11].

This work was later extended by Furmans [7], who introduced a discrete time framework for modeling material-handling systems as networks of interconnected stochastic elements, including queueing, split, and merge nodes. For the superposition of independent input streams, Furmans approximates the merged point process as a renewal process. The approximation is based on the forward recurrence times of the incoming renewal processes, with the next arrival determined by their minimum. Furmans also notes that this approximation can become inaccurate when dependencies between incoming streams are ignored, particularly when the streams originate from a deterministic split and the intermediate sojourn times exhibit little variability.

Furmans' analytical merge approximates the superposition of incoming streams as a renewal process based on their marginal inter-arrival time distributions. Although Furmans identifies dependencies between the incoming streams as a potential source of error, the conditions under which these dependencies cause the approximation to become inaccurate are not quantified.

This paper investigates this limitation for input streams with zero relative temporal offset. In particular, it examines cases in which only a restricted set of relative arrival positions can occur, while the renewal approximation assigns probability to additional configurations. Based on this analysis, an extension of Furmans' merge method is developed that removes impossible configurations and renormalizes the resulting distribution.

3 Method

This section presents the methods used to analyze the stochastic merge. First, the problem setting and the limitation of the renewal-based merge are introduced. Subsequently, Furmans' analytical merge and the proposed SAFE - Senger Altmann Furmans Extension are described, followed by the numerical merge and the discrete time simulation used as a benchmark.

3.1 Problem setting

A merge node combines two or more incoming discrete time streams into a single output stream that becomes the input to a downstream queueing element (see Figure 1). Let $(X_i(t))$ denote the number of arrivals from stream i in time step t . An example of two incoming streams is shown in Figure 2(a). The inter-arrival time describes the time between two consecutive arrivals, whereas the *forward recurrence time* describes the time from an observation point to the next arrival. The merge of arrivals in the process is given by

$$X_M(t) = \sum_{i=1}^m X_i(t).$$

Conceptually, the merge is obtained by superimposing the arrival epochs of all incoming streams on a common time axis, as illustrated in Figure 2(b). The inter-arrival times of the merged stream are then determined from the ordered arrival epochs. If several arrivals occur in the same time step, they are retained as simultaneous arrivals, resulting in an inter-arrival time of zero between them.

In a decomposed queueing network, however, the incoming streams are typically not known through their complete arrival sequences. Instead, they are represented by their marginal inter-departure time distributions. Furmans [7] therefore approximates the merged stream as a renewal process based on the forward recurrence times of the individual input streams.

3.2 Analytical renewal-based merge

Following [7], Furmans' merge proceeds in three steps:

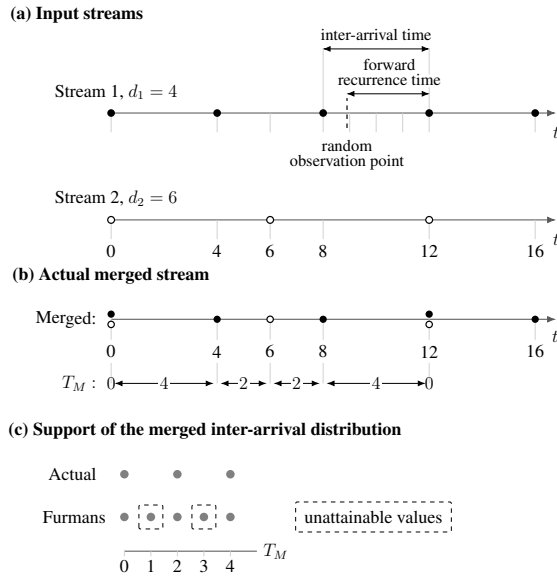


Figure 2: Illustration of the merge of two deterministic input streams with inter-arrival times of $d_1 = 4$ and $d_2 = 6$ time units. (a) Input streams with the inter-arrival time, a random observation point, and the corresponding forward recurrence time. (b) Superposition of both streams and resulting merged inter-arrival times T_M . (c) Support of the actual merged inter-arrival time distribution compared with Furmans' renewal approximation, which assigns additional probability mass to the unattainable values 1 and 3.

1. the forward recurrence time distribution $\alpha_{i \rightarrow h}$ is derived from each marginal inter-arrival distribution p_i (described in 3.2.1),
2. the forward recurrence time of the merged stream is obtained as the distribution of the minimum over all component recurrence times (described in 3.2.2), and
3. the inter-arrival distribution of the merged stream is reconstructed (described in 3.2.3).

3.2.1 Forward Recurrence Time of the Input Streams

The forward recurrence time distribution of stream i is obtained from its marginal inter-arrival distribution as

$$\alpha_{i,j} = \frac{1}{\mathbb{E}[A_i]} \left(1 - \sum_{k=0}^j p_i(k) \right), \quad (1)$$

where $\alpha_{i,j}$ denotes the probability of a forward recurrence time of j discrete time units, and $p_i(k) = \Pr(A_i = k)$ denotes the probability that the inter-arrival time A_i of stream i equals k discrete time units.

Hence, the forward recurrence time distribution is a stationary marginal description of the residual time of the corresponding renewal process. For example, for a deterministic inter-arrival time $A_i = n$, Eq. (1) yields

$$\alpha_{i,j} = \frac{1}{n}, \quad j = 0, \dots, n-1. \quad (2)$$

Thus, all n possible residual positions within the deterministic renewal interval receive equal probability.

For an individual renewal process, this stationary forward recurrence time distribution provides a valid marginal description. When several streams are merged, however, dependencies between their forward recurrence times may arise due to their relative temporal alignment. Since no information on the offsets between the individual streams is available, we assume these offsets to be uniformly distributed. If the streams have a fixed temporal relation, not every combination of their marginal forward recurrence times can occur at the same observation time.

Consider two deterministic streams with inter-arrival times of 4 and 6 time units and zero relative temporal offset. Their arrival epochs are

$$0, 4, 8, 12, \dots \quad (3)$$

and

$$0, 6, 12, 18, \dots \quad (4)$$

According to Eq. (1), the marginal forward recurrence time distributions assign equal probability to $\{0, 1, 2, 3\}$ for the first stream and $\{0, 1, 2, 3, 4, 5\}$ for the second stream. These marginal distributions are individually correct. However, because both streams are observed at the same point in time, their forward recurrence times cannot take arbitrary combinations.

For example, at observation time $t = 2$, the forward recurrence times are 2 and 4 time units. In contrast, a combination such as $(0, 1)$ cannot occur for these two streams with zero relative temporal offset. Furmans' renewal-based merge does not preserve this temporal relation when combining the marginal forward recurrence time distributions and may therefore assign probability to configurations that are not attainable in the actual merged process.

As illustrated in Figure 2(c), this results in additional support values in the merged inter-arrival time distribution. For the example above, the actual merged process contains the inter-arrival times 0, 2, and 4, whereas the renewal approximation also assigns probability to the unattainable values 1 and 3.

3.2.2 Forward Recurrence Time of the Merged Stream

For two incoming streams, the time to the next arrival of the merged stream is the minimum of the two forward recur-

rence times. Assuming independent input streams, we obtain:

$$\begin{aligned}\alpha_{h,j} &= \alpha_{1,j} \left(1 - \sum_{k=0}^j \alpha_{2,k} \right) \\ &+ \alpha_{2,j} \left(1 - \sum_{k=0}^j \alpha_{1,k} \right) \\ &+ \alpha_{1,j} \alpha_{2,j}.\end{aligned}\quad (5)$$

3.2.3 Inter-Arrival Time Distribution of the Merged Stream

To reconstruct the inter-arrival time distribution, the mean inter-arrival time of the merged stream is first obtained from the superposed arrival rate:

$$\lambda_h = \lambda_1 + \lambda_2 = \frac{1}{\mathbb{E}[A_1]} + \frac{1}{\mathbb{E}[A_2]}, \quad (6)$$

and therefore

$$\mathbb{E}[A_h] = \frac{1}{\lambda_h}. \quad (7)$$

Using the inverse relation of Eq. (1), the inter-arrival probabilities of the merged stream are then reconstructed as

$$p_h(j) = \mathbb{E}[A_h] (\alpha_{h,j-1} - \alpha_{h,j}), \quad j = 1, \dots, j_{\max} - 1, \quad (8)$$

with

$$p_h(j_{\max}) = \mathbb{E}[A_h] \alpha_{h,j_{\max}-1}, \quad (9)$$

and

$$p_h(0) = 1 - \sum_{j=1}^{j_{\max}} p_h(j). \quad (10)$$

For more than two incoming streams, the procedure is applied iteratively. After merging two streams, the resulting inter-arrival time distribution is used as an input distribution for the merge with the next stream until all m streams have been combined.

3.3 SAFE Merge

The SAFE merge extends Furmans' renewal-based merge for input streams whose inter-arrival times are restricted to a discrete time grid with zero relative temporal offset. Furman's original approach combines the marginal forward recurrence time distributions of the individual input streams independently. Since no information about the relative offsets between the streams is available, these offsets cannot be modeled directly. To address this lack of information, Furman's approach follows the principle of insufficient reason (Laplace's principle) [12] and considers the forward recurrence times from a uniformly distributed observation point, such that no particular observation point is favored over another. For streams whose arrivals are restricted to

a discrete arrival grid, however, not every combination of forward recurrence times can occur at the same observation time. Assuming that both processes start at the same time, the SAFE merge therefore removes inadmissible combinations, normalizes the remaining probability mass, and subsequently applies the minimum operation of the original Furmans merge.

Throughout this work, all input streams are assumed to have zero relative temporal offset, i.e., they share a common reference arrival at time $t = 0$.

3.3.1 Admissible Forward Recurrence Times

Consider two input streams A_1 and A_2 , with sets of possible inter-arrival times given by

$$A_1 = a_{1,1}, a_{1,2}, \dots, a_{1,n_1} \quad (11)$$

and

$$A_2 = a_{2,1}, a_{2,2}, \dots, a_{2,n_2}. \quad (12)$$

For each stream, the GCD of its possible inter-arrival times is determined as

$$g_1 = \gcd(a_{1,1}, a_{1,2}, \dots, a_{1,n_1}) \quad (13)$$

and

$$g_2 = \gcd(a_{2,1}, a_{2,2}, \dots, a_{2,n_2}). \quad (14)$$

The GCD of both streams is then obtained by

$$g = \gcd(g_1, g_2). \quad (15)$$

Thus, g defines a common temporal grid for the possible inter-arrival times of both input streams.

For example, consider two deterministic input streams with inter-arrival times 4 and 6. Their GCD are

$$g_1 = 4, \quad g_2 = 6, \quad (16)$$

and therefore

$$\gcd(g_1, g_2) = 2. \quad (17)$$

3.3.2 Observation-Time-Conditioned Forward Recurrence Time Distributions

Since g defines the common arrival-grid spacing of the two input streams, the SAFE extension restricts the support of the forward recurrence time distributions to grid positions that are integer multiples of g . Thus, only support points j satisfying

$$j \equiv 0 \pmod{g} \quad (18)$$

are retained.

Let $\alpha_{i,j}$ denote the probability mass assigned to support point j of the forward recurrence time distribution of stream i , obtained from Furmans' original formulation (see

Section 3.2.1). For a given observation time t , all probability values that do not satisfy Eq. (18) are removed. The remaining probability mass is subsequently normalized. The resulting distribution is

$$\alpha_{i,j}^{(t)} = \frac{\alpha_{i,j} \mathbf{1}_{\{j \bmod g=0\}}}{\sum_{\substack{k \geq 0 \\ k \bmod g=0}} \alpha_{i,k}}. \quad (19)$$

Here, the indicator function is defined as

$$\mathbf{1}_{\{j \bmod g=0\}} = \begin{cases} 1, & \text{if } j \bmod g = 0, \\ 0, & \text{otherwise.} \end{cases} \quad (20)$$

The indicator function removes forward recurrence times that cannot occur, while the denominator normalizes the remaining probabilities such that

$$\sum_{j \geq 0} \alpha_{i,j}^{(t)} = 1. \quad (21)$$

From this point onward, the original Furmans' procedure is applied as described in Sections 3.2.2 and 3.2.3: the conditioned forward recurrence time distributions are merged using the minimum operation, and the resulting merged forward recurrence time distribution is transformed back into the inter-arrival time distribution.

3.4 Numerical Merge

The numerical merge generates the incoming renewal streams explicitly over a finite number of arrival events H and combines them according to their arrival times. Each stream is initialized at time $t = 0$ and subsequent inter-arrival times are sampled from its corresponding marginal distribution.

At each step, the stream with the earliest next arrival is selected and advanced by a newly sampled inter-arrival time. The next arrival of the merged stream is then given by the earliest arrival among all streams. The difference between two consecutive merged arrival times defines the merged inter-arrival time,

$$A_{M,k} = T_{k+1} - T_k. \quad (22)$$

If two streams arrive at the same time, the corresponding merged inter-arrival time is zero.

After generating H merged inter-arrival times, the merged distribution is estimated from their relative frequencies,

$$\hat{p}_M(a) = \frac{1}{H} \sum_{k=1}^H \mathbf{1}_{\{A_{M,k}=a\}}. \quad (23)$$

The numerical method therefore preserves the temporal positions of the generated renewal streams during the merge, while the individual streams are sampled independently from their marginal inter-arrival time distributions.

3.5 Simulation benchmark

A discrete time simulation is used as the benchmark because it preserves the event history and the job-level dependence between streams. The simulation operates in fixed time steps. In each time step, external arrivals are generated according to the specified arrival process, jobs enter the corresponding queue if capacity is available, service times are sampled from the assigned discrete service time distributions, and completed jobs are routed to downstream nodes. For the merge experiments, the simulation records the arrival epochs of all incoming streams at the merge node. The merged event sequence is then constructed by superposing these recorded arrival epochs. If multiple streams arrive in the same time step, the resulting merged sequence includes zero inter-arrival times for the additional simultaneous arrivals. The empirical merged inter-arrival time distribution is calculated as

$$\hat{p}_M^{\text{sim}}(a) = \frac{1}{n} \sum_{\ell=1}^n \mathbf{1}_{\{A_{M,\ell}^{\text{sim}} = a\}},$$

where $A_{M,\ell}^{\text{sim}}$ denotes the ℓ -th observed merged inter-arrival time and n is the number of observations remaining after removal of the warm-up period, which is determined using the MSER-5 method [13].

4 Results

All experiments consider the merge of exactly two input streams. Each input stream is assigned a squared coefficient of variation from $c^2 \in \{0, 0.25, 0.5, 1, 1.5, 2, 2.5, 3, 4, 5, 10\}$, resulting in ten unique combinations of c^2 values. For each combination, both input streams share the same gcd , which is varied over $gcd \in \{1, 2, 3, 4, 5, 10\}$. The resulting inter-arrival time distributions are evaluated using the Wasserstein distance to a discrete time simulation benchmark. The simulation length was selected based on the convergence of the inter-arrival time distribution. To assess convergence, the Wasserstein distance is evaluated after every additional 100,000 samples by comparing the current inter-arrival time distribution with that obtained at the preceding evaluation. The simulation length is considered sufficient once the relative width of the 95% confidence interval of the Wasserstein distance falls below 1%, indicating that the uncertainty in the estimated distance has become sufficiently small. The choice of the merging approach depends on the available information about the temporal alignment of the arrival streams. If the temporal offset between the streams is known, their relative alignment can be explicitly accounted for and the SAFE merge is applied. In particular, an offset of zero corresponds to synchronized streams. If no information about the temporal offset is available, the original Furmans' merging approach is used, implicitly assuming a uniformly distributed relative offset between the streams.

4.1 Accuracy of the Merge Methods

The first experiment evaluates how the structure of the input distributions affects the accuracy of the different merge methods. In particular, the analysis focuses on the GCD. For $gcd = 1$, no restriction on the discrete time grid is present, whereas $g > 1$ restricts the possible inter-arrival times and therefore the possible forward recurrence times. Figure 3

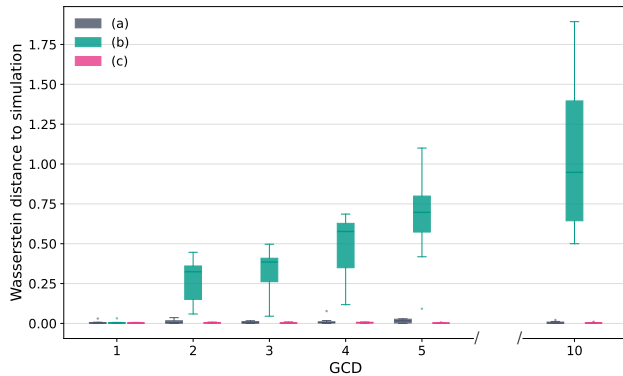


Figure 3: Wasserstein distance to the simulation benchmark for (a) the SAFE merge, (b) Furmans' original renewal-based merge [7], and (c) the numerical merge, evaluated for different GCDs and c^2 values of the input inter-arrival time distributions.

compares the Wasserstein distance of the inter-arrival time distributions obtained with the simulation described in 3.5 and the three merge methods across different values of the GCD. The SAFE method and the numerical merge remain close to the simulation across all considered configurations, with Wasserstein distances close to zero. In contrast, Furmans' original merge shows larger deviations, which generally increase as the GCD increases. The variability of these deviations also becomes more pronounced for larger GCD values. Furmans' approximation therefore provides accurate results when the input streams do not exhibit the grid restrictions addressed by SAFE. Its error increases for configurations with $gcd > 1$, where the independent forward recurrence time representation includes values that are not compatible with the temporal structure of the input streams.

The SAFE merge removes these impossible values and therefore substantially reduces the resulting error. The numerical merge also reproduces the simulation results closely, since the inter-arrival sequences are generated explicitly over time.

Analytical outliers can arise from sampling variability in the simulation rather than from inaccuracies in the analytical solution. Since the simulation is based on a finite number of samples, deviations from the theoretical distribution are unavoidable.

4.2 Convergence of the Numerical Merge

The numerical merge can reproduce the merged inter-arrival time distribution without relying on the renewal approximation. Its accuracy, however, depends on the number of generated samples H . Figure 4 shows the convergence of the numerical merge toward the analytical reference for three different merge configurations. In all three cases, the analytical model provides the exact distribution. However, the number of samples required to accurately reproduce this distribution differs substantially between the configurations. The configurations differ in the magnitude of the smallest non-zero probabilities occurring in the underlying distributions. For configurations (a), (b), and (c), the smallest probabilities are approximately 0.5, 0.01, and 10^{-5} , respectively. As these probabilities decrease, increasingly large sample sizes are required to observe the corresponding low-probability events sufficiently often and to accurately reproduce their contribution to the overall distribution. Consequently, configurations containing rare events exhibit considerably slower convergence. While some configurations reach a Wasserstein distance close to zero at approximately 10^7 samples, configurations involving very small probabilities may require up to 10^8 samples. The numerical merge is therefore able to obtain accurate results for all considered configurations, but the required sampling horizon can become very large, particularly when the underlying distributions contain low-probability events. This results in a substantial computational effort and limits the suitability of the numerical merge for repeated evaluations in larger queueing networks. In contrast, the analytical computation can be performed efficiently without requiring extensive sampling. The SAFE merge similarly provides an analytical approximation and therefore avoids the potentially very large sampling horizons H required by the numerical approach.

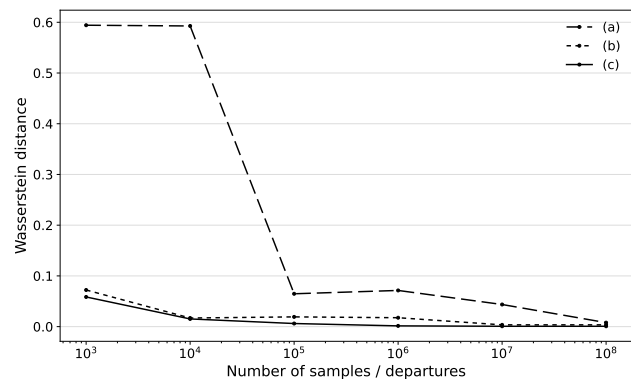


Figure 4: Convergence of the numerical merge toward the exact Furmans reference, measured by the Wasserstein distance for increasing sampling horizons H across three merge configurations.

5 Conclusion

This paper investigates the limitations of Furmans' renewal-based merge for discrete time stochastic flow networks and proposes the SAFE merge as an analytical extension. The results show that Furmans' approximation becomes increasingly inaccurate when the input streams are restricted to a common discrete arrival grid with a GCD greater than one. By removing inadmissible forward recurrence time values and renormalizing the remaining probability mass, SAFE substantially reduces this error and remains close to the simulation benchmark across the investigated configurations.

The numerical merge also provides accurate results, but its convergence can require very large sampling horizons, particularly when low-probability events are present. In the investigated cases, up to approximately 10^8 samples were required to closely reproduce the reference distribution. SAFE therefore provides a computationally efficient analytical alternative for cases in which Furmans' original approximation is insufficient.

From a material handling perspective, SAFE extends the range of flow configurations that can be represented accurately within Furmans' discrete-time modeling framework. This is particularly relevant for automated storage and retrieval systems, where storage and retrieval requests form two input streams to a common storage and retrieval machine. If these streams are temporally synchronized, their relative arrival positions are restricted and may not be captured correctly by an independent renewal-based merge. For zero relative temporal offset, SAFE provides a more accurate representation of the merged arrival process and therefore improves the analytical evaluation of the downstream queueing system.

The current formulation assumes zero relative temporal offset between the input streams. Extending the method to non-zero offsets remains a topic for future work, particularly for material handling systems in which storage and retrieval streams are not synchronized.

REFERENCES

- [1] M. Schleyer, "Discrete time analysis of batch processes in material flow systems," Ph.D. dissertation, Universität Karlsruhe (TH), 2007. [Online]. Available: <https://doi.org/10.5445/KSP/1000006694>
- [2] E. Özden, *Discrete time analysis of consolidated transport processes*. KIT Scientific Publishing, 2012, vol. 77. [Online]. Available: <https://doi.org/10.5445/KSP/1000025608>
- [3] J. M. Matzka, *Discrete time analysis of multi-server queueing systems in material handling and service*. KIT Scientific Publishing, 2011, vol. 75. [Online]. Available: <https://doi.org/10.5445/KSP/1000023289>
- [4] K. Furmans, T. Schmidt, I. Meinhardt, M. Epp, and M. Rimmele, "Analytical computation of sojourn time distributions in large-scale conveyer systems," *SMMSO 2015*, pp. 75–82, 2015. [Online]. Available: <https://publikationen.bibliothek.kit.edu/1000049476>
- [5] M. Epp, *Performance evaluation of shuttle-based storage and retrieval systems using discrete-time queueing network models*. KIT Scientific Publishing, 2018, vol. 90. [Online]. Available: <https://doi.org/10.5445/KSP/1000077211>
- [6] U. Mohring, M. Baumann, and K. Furmans, "Discrete-Time Analysis of Levelled Order Release and Staffing in Order Picking Systems," *Logistics Research*, vol. 13, no. 9, 2020. [Online]. Available: https://doi.org/10.23773/2020_9
- [7] K. Furmans, "A framework of stochastic finite elements for models of material handling systems," *Progress in material handling research*, vol. 8, 2004. [Online]. Available: <https://publikationen.bibliothek.kit.edu/1000021730>
- [8] H. Bruneel, "Performance of discrete-time queueing systems," *Computers & Operations Research*, vol. 20, no. 3, pp. 303–320, 1993. [Online]. Available: [https://doi.org/10.1016/0305-0548\(93\)90006-5](https://doi.org/10.1016/0305-0548(93)90006-5)
- [9] W. K. Grassmann and J. L. Jain, "Numerical solutions of the waiting time distribution and idle time distribution of the arithmetic GI/G/1 queue," *Operations Research*, vol. 37, no. 1, pp. 141–150, 1989. [Online]. Available: <https://doi.org/10.1287/opre.37.1.141>
- [10] J. L. Jain and W. K. Grassmann, "Numerical solution for the departure process from the GI/G/1 queue," *Computers & Operations Research*, vol. 15, no. 3, pp. 293–296, 1988. [Online]. Available: [https://doi.org/10.1016/0305-0548\(88\)90042-1](https://doi.org/10.1016/0305-0548(88)90042-1)
- [11] K. Furmans and A. Zillus, "Modeling independent production buffers in discrete time queueing networks," *Proceedings of CIMAT'96, Grenoble*, pp. 275–280, 1996. [Online]. Available: <https://doi.org/10.5445/IR/279796>

- [12] I. Gilboa, “The Principle of Indifference,” in *Theory of Decision under Uncertainty*, ser. Econometric Society Monographs. Cambridge: Cambridge University Press, 2009, pp. 14–19. [Online]. Available: <https://doi.org/10.1017/CBO9780511840203.005>
- [13] K. Hoad and S. Robinson, “Implementing MSER-5 in commercial simulation software and its wider implications,” in *Proceedings of the 2011 Winter Simulation Conference*, S. Jain, R. R. Creasey, J. Himmelspach, K. P. White, and M. Fu, Eds. Piscataway, NJ: IEEE, 2011, pp. 495–503. [Online]. Available: <https://doi.org/10.1109/WSC.2011.6147779>

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